The Hills Grammar School

Founded 1982

2013
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen.
  Black pen is preferred.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper which may be detached and used throughout the paper.
- In Questions 11-16, show relevant mathematical reasoning and/or working.

Total marks – 100

Section I

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice Answer Sheet provided.
- Allow about 15 minutes for this Section.

Section II

90 marks

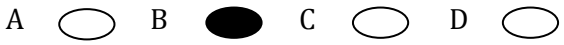
- Attempt Questions 11-16
- Answer in the writing booklets provided.
- Start a new booklet for each question.
- Allow about 2 hours and 45 minutes for this Section.

Section I
Total marks (10)
Attempt Questions 1-10
Allow about 15 minutes for this section

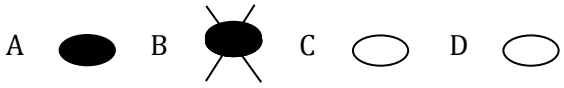
Use the Multiple Choice Answer Sheet provided.
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Example: 2 + 4 = ?

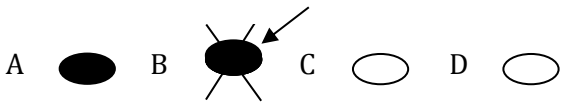
(A) 2
(B) 6
(C) 8
(D) 9



If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:



1 The gradient of the line $2x-3y+7=0$ is:

(A) 2

(B) $\frac{2}{3}$

(C) $-\frac{2}{3}$

(D) $\frac{3}{2}$

2 The conditions for the expression $ax^2+bx+c=0$ to be positive definite are:

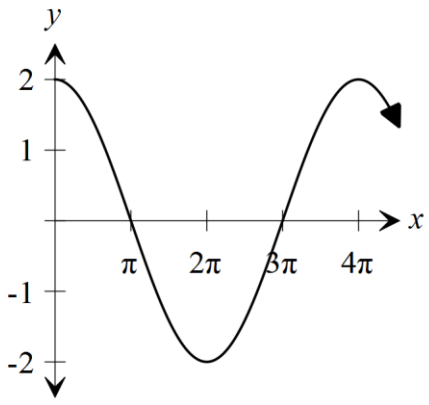
(A) $a>0$ and $\Delta>0$

(B) $c>0$ and $\Delta>0$

(C) $c>0$ and $\Delta<0$

(D) $a>0$ and $\Delta<0$

3



NOT TO SCALE

Which equation would best describe the trigonometric graph shown above?

(A) $y=\frac{1}{2}\sin 2x$

(B) $y=2\sin 2x$

(C) $y=2\cos 2x$

(D) $y=2\cos\frac{x}{2}$

4 What is the domain and range of the function $y = \sqrt{9 - x^2}$?

(A) Domain $-3 \le x \le 3$ Range $0 \le y \le 3$

(B) Domain $-3 \le x \le 3$ Range $-3 \le y \le 3$

(C) Domain $0 \le x \le 3$ Range $-3 \le y \le 3$

(D) Domain $0 \le x \le 3$ Range $0 \le y \le 3$

5 What is the value of $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$?

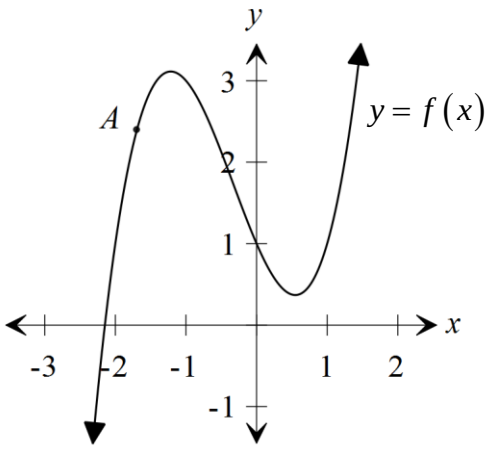
(A) 0

(B) undefined

(C) 4

(D) 6

6 What properties exist at the point *A* on the graph of $y = f(x)$ shown below?



NOT TO SCALE

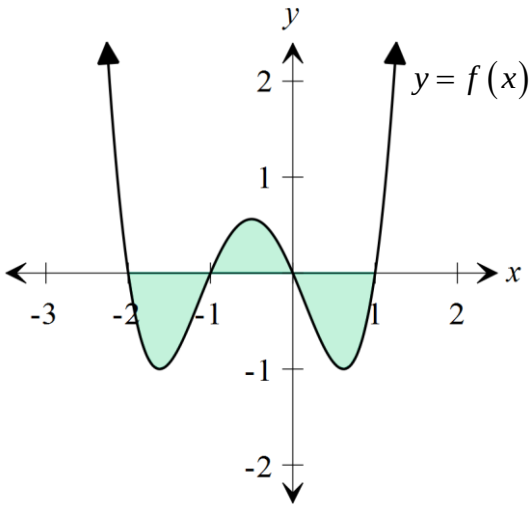
(A) $f'(x) > 0$ and $f''(x) > 0$

(B) $f'(x) > 0$ and $f''(x) < 0$

(C) $f'(x) < 0$ and $f''(x) > 0$

(D) $f'(x) < 0$ and $f''(x) < 0$

7 The diagram below shows the graph of $y = f(x)$. Which formula would give the best approximation of the shaded area between $y = f(x)$ and the x axis?



NOT TO SCALE

(A) $A = \int_{-2}^{1} f(x)\,dx$

(B) $A = \left| \int_{-2}^{0} f(x)\,dx \right| + \int_{0}^{1} f(x)\,dx$

(C) $A = 2\int_{-2}^{-1} f(x)\,dx + \int_{-1}^{0} f(x)\,dx$

(D) $A = \int_{-1}^{0} f(x)\,dx + 2\left| \int_{-2}^{-1} f(x)\,dx \right|$

8 What is the value of $\int_{1}^{\sqrt{3}} \frac{2dx}{x}$ in simplest form?

(A) $\ln 2$

(B) $2\ln\sqrt{3} - 2\ln 1$

(C) $\left(\ln\sqrt{3}\right)^2$

(D) $\ln 3$

9 Ten milligrams of caffeine is placed in hot water and begins to dissolve. After t minutes the amount A mg of undissolved caffeine is given by $A = 10e^{-kt}$. What is the value of k given that $A = 3.6$ and $t = 5$?

(A) -0.717

(B) -0.204

(C) 0.204

(D) 0.717

10 In the arithmetic sequence $1, 3+x, \ldots$ what will be the 20^{th} term?

(A) $(3+x)^{20}$

(B) $(3+x)^{19}$

(C) $19x+39$

(D) $20x+41$

End of Section I

Section II

Total marks (90)
Attempt Questions 11-16
Start a new booklet for each Question
Allow about 2 hours and 45 minutes for this section

START A NEW BOOKLET
Question 11 (15 marks) **Marks**

(a) Evaluate $\sqrt{\dfrac{\pi}{0.12\times 3.16}}$ correct to 2 significant figures. **2**

(b) Express 225° in radians in terms of π. **1**

(c) Rationalise the denominator of $\dfrac{2}{\sqrt{3}+1}$. Give your answer in simplest form. **2**

(d) Simplify $\dfrac{3x-9}{x^2-9}$. **2**

(e) Find the exact values of θ such that $\tan\theta = \dfrac{-1}{\sqrt{3}}$, where $0 \le \theta \le 2\pi$. **2**

(f) Solve $9^x = \dfrac{1}{27}$. **2**

(g) Find the coordinates of the focus of the parabola $x^2 = 8(y+1)$. **2**

(h) Solve $|2x-1| > 8$. **2**

End of Question 11

START A NEW BOOKLET
Question 12 (15 marks) **Marks**

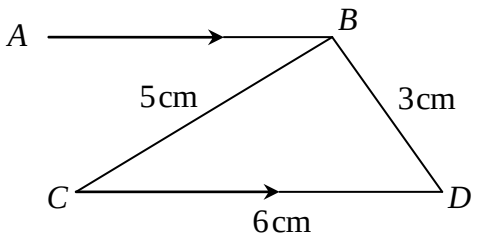
(a) Differentiate with respect to x

(i) $\sqrt{x}$ **1**

(ii) $\tan\left(e^{2x}\right)$ **2**

(iii) 4^x. **2**

(b)



NOT TO SCALE

In the diagram, AB is parallel to CD, CB is 5 cm, CD is 6 cm, BD is 3 cm.

(i) Use the cosine rule to find the size of $\angle BDC$, to the nearest degree. **2**

(ii) Hence find the size of $\angle ABD$, giving reasons. **2**

(c) Solve $\log_2\left(5x+1\right)=4$. **2**

(d) Find:

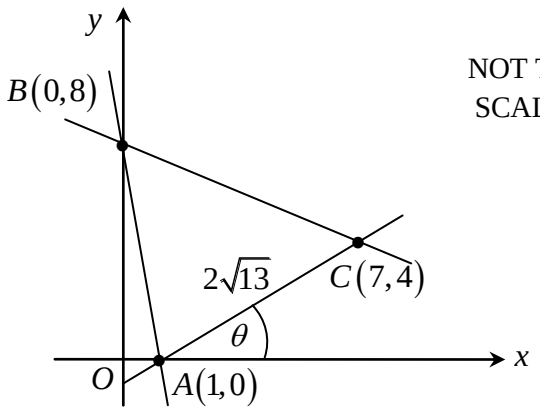
(i) $\int\left(2x-1\right)^3\,dx$ **2**

(ii) $\int_0^{\ln 2} e^{-x}dx$. **2**

End of Question 12

START A NEW BOOKLET
Question 13 (15 marks) **Marks**

(a) The points *A*, *B* and *C* have coordinates $(1,0)$, $(0,8)$ and $(7,4)$, as shown in the diagram below. The angle between the line *AC* and the *x* axis is θ. $AC = 2\sqrt{13}$.



(i) Find the gradient of the line *AC*. **1**

(ii) Calculate the size of angle θ to the nearest degree. **1**

(iii) Find the equation of the line *AC*. **1**

(iv) Find the coordinates of *D*, the midpoint of *AC*. **1**

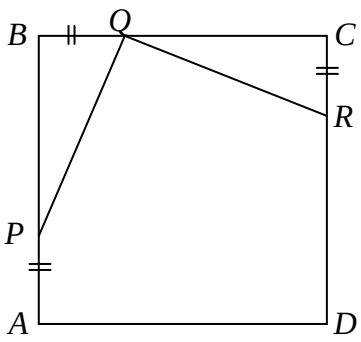
(v) Show that *AC* is perpendicular to *BD*. **1**

(vi) Find the area of ΔABC. **2**

(vii) Write down the coordinates of a point *E* such that *ABCE* is a rhombus. **1**

Question 13 continues on the next page

(b) In the diagram *ABCD* is a square. The points *P*, *Q* and *R* lie on *AB*, *BC* and *CD* respectively such that $AP = BQ = CR$.

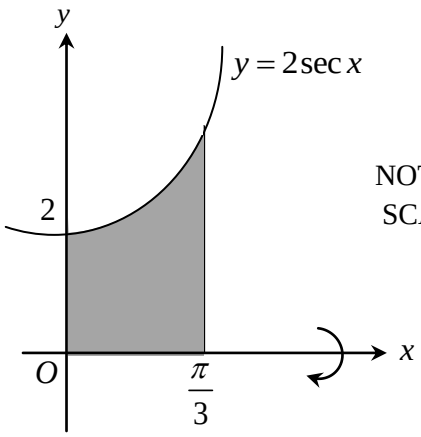


NOT TO SCALE

(i) Prove that ΔPBQ and ΔCQR are congruent. **2**

(ii) Prove that $\angle PQR$ is a right angle. **2**

(c) In the diagram the shaded region is bounded by the curve $y = 2\sec x$, the coordinate axes and the line $x = \frac{\pi}{3}$. The shaded region is rotated about the *x*-axis.



NOT TO SCALE

Calculate the exact volume of the solid of revolution formed. **3**

End of Question 13

START A NEW BOOKLET
Question 14 (15 marks) **Marks**

(a) If α and β are the roots of $2x^2 - 3x - 4 = 0$, find the value of

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

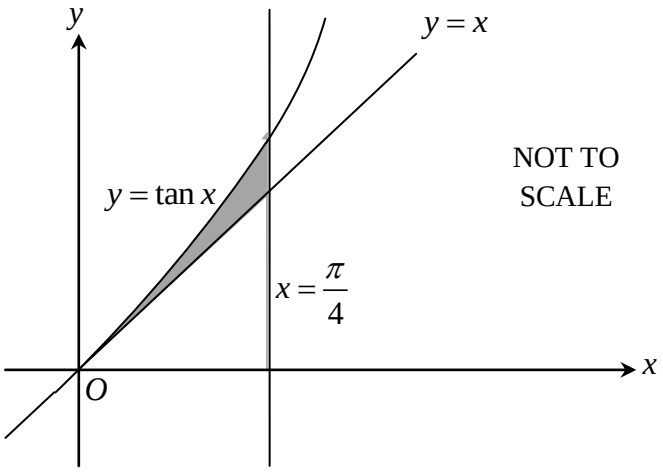
(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **1**

(iv) $\alpha^2 + \beta^2$. **2**

(b) Prove that $\sec^2 x + \sec x \tan x = \dfrac{1}{1 - \sin x}$ **2**

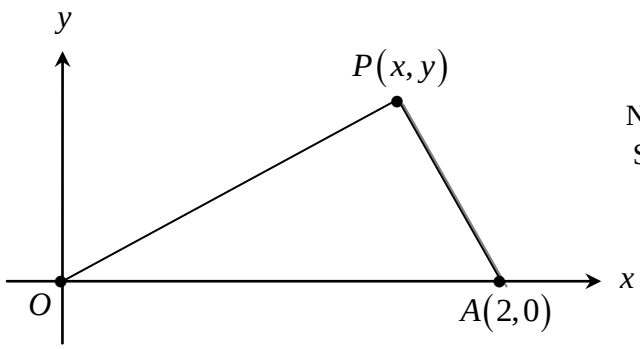
(c) (i) Show that $\dfrac{d}{dx}\log_e(\cos x) = -\tan x$. **1**

(ii) The shaded region in the diagram is bounded by the curve $y = \tan x$ and the lines $y = x$ and $x = \dfrac{\pi}{4}$. Using the result from part (i), or otherwise, find the area of the shaded region. **3**



Question 14 continues on the next page

(d)



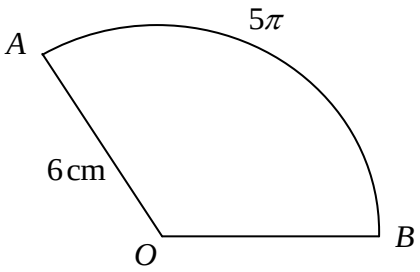
(i) Show that the equation of the locus of all points P such that OP is perpendicular to AP is $x^2 - 2x + y^2 = 0$. **2**

(ii) Hence determine the centre and radius of the locus in part (i). **2**

End of Question 14

START A NEW BOOKLET
Question 15 (15 marks) **Marks**

(a)



NOT TO SCALE

AOB is a sector of a circle, centre *O* and radius 6 cm . The length of the arc *AB* is 5π .
Calculate the area of sector *AOB.* **2**

(b) A ball is dropped from a height of 4 metres onto a hard floor and bounces. After each bounce, the maximum height reached by the ball is 75% of the previous maximum height. Thus after it first hits the floor, it reaches a height of 3 metres before falling again, and after the second bounce it reaches a height of 2.25 metres before falling again.

(i) What kind of sequence is formed by successive maximum heights? **1**

(ii) What is the total distance travelled by the ball from the time it was first dropped until it eventually comes to rest on the floor? **2**

(c) (i) Sketch the graph of $y = \log_e x$ and shade the region defined by $\int_1^5 \log_e x \, dx$ **2**

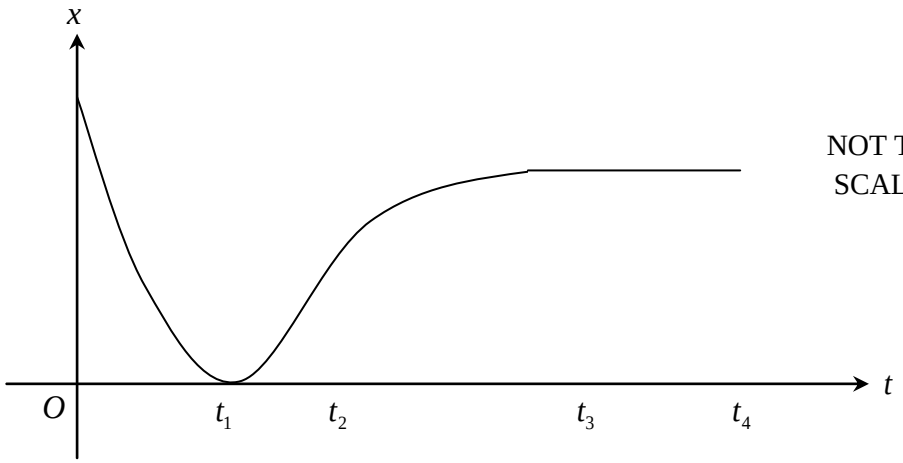
(ii) Use Simpson's rule with three function values to estimate $\int_1^5 \log_e x \, dx$ **2**

Give your answer correct to 2 decimal places.

(d) The curve $y = ax^3 - 9x^2 + b$ has a minimum turning point at $(3, -12)$.
Find the values of a and b. **3**

Question 15 continues on the next page

(e) A particle moves in a straight line and the graph shows the distance x of the particle from a fixed point at time t.



(i) What is the velocity at $t = t_3$? **1**

(ii) Sketch the graph of velocity v as a function of time. **2**

End of Question 15

START A NEW BOOKLET
Question 16 (15 marks) **Marks**

(a) Selina invests $400 in a bank account at the beginning of each month for 8 years. Interest is to be paid at a rate of 6% per annum compounded monthly.

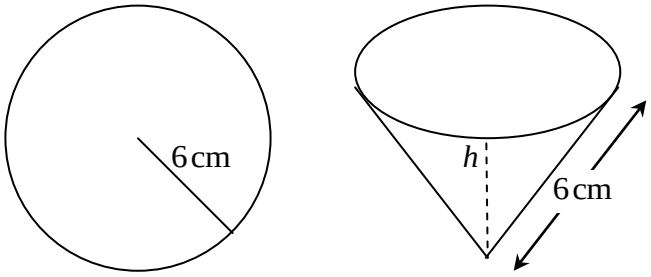
(i) Find the amount in the account at the end of the first month. **1**

(ii) Show that the total value of her investment at the end of n years is given by $\$400\left(1.005+1.005^2+1.005^3+\ldots 1.005^{12n}\right)$ **1**

(iii) Find the final value of Selina's investment at the end of the 8 years. Give your answer correct to the nearest dollar. **3**

(iv) What **single** investment at the beginning of the 8 years, with interest compounded monthly, would achieve the same final value? Answer to the nearest dollar. **2**

(b) A circular filter paper of radius $6\,\text{cm}$ is cut once along the radius and then each edge formed by this cut is overlapped to make a conical filter with height *h* and radius *r*.



NOT TO SCALE

(i) Show that the volume, V, of the cone is $\frac{1}{3}\pi r^2\sqrt{36-r^2}$ where r is the base radius. **2**

(ii) Show that $\frac{dV}{dr}=\frac{2\pi r\sqrt{36-r^2}}{3}-\frac{\pi r^3}{3\sqrt{36-r^2}}$ **3**

(iii) Find the maximum volume of the cone and the corresponding radius in exact form. **3**

End of Examination Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \qquad x > 0$

Student Number

Multiple Choice Answer Sheet

Section I
Total marks (10)
Attempt Questions 1-10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

1	(A)	(B)	(C)	(D)
2	(A)	(B)	(C)	(D)
3	(A)	(B)	(C)	(D)
4	(A)	(B)	(C)	(D)
5	(A)	(B)	(C)	(D)
6	(A)	(B)	(C)	(D)
7	(A)	(B)	(C)	(D)
8	(A)	(B)	(C)	(D)
9	(A)	(B)	(C)	(D)
10	(A)	(B)	(C)	(D)

Student Number

The Hills Grammar School

Founded 1982

2013
Higher School Certificate
Trial Examination

Mathematics

Question	Algebra and Number	Differential Calculus	Functions	Geometry	Integral Calculus	Trigonometry	Total
1-10	10 **/1**	5, 6, 9 **/3**	1, 2, 4 **/3**		7, 8 **/2**	3 **/1**	/10
11	(a), (c), (d), (f), (h) **/10**		(g) **/2**			(b), (e) **/3**	/15
12	(c) **/2**	(a) **/5**		(b)(ii) **/2**	(d) **/4**	(b)(i) **/2**	/15
13			(a) **/8**	(b) **/4**	(c) **/3**		/15
14		(c)(i) **/1**	(a), (d) **/9**		(c)(ii) **/3**	(b) **/2**	/15
15	(b) **/3**	(d), (e) **/6**			(c) **/4**	(a) **/2**	/15
16	(a) **/7**	(b) **/8**					/15
Total	**/23**	**/23**	**/22**	**/6**	**/16**	**/10**	**/100**

THGS Mathematics
THSC 2013 Solutions

Multiple Choice

1. B 2. D 3. D 4. A 5. C 6. B
7. D 8. D 9. C 10. C

Question 11

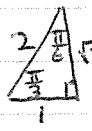
(a) $2.8783\ldots$ ✓
2.9 (2 sig figs) ✓

(b) $225 \times \frac{\pi}{180} = \frac{5\pi}{4}$ ✓

(c) $\frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{2\sqrt{3}-2}{2}$ ✓
$= \sqrt{3}-1$ ✓

(d) $\frac{3(x-3)}{(x-3)(x+3)} = \frac{3}{x+3}$ ✓

(e) S | A
T | C ✓



aux $\angle = \frac{\pi}{6}$ ✓
$\theta = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$
$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$ ✓

(f) $3^{2x} = 3^{-3}$ ✓
$\therefore 2x = -3$
$x = -\frac{3}{2}$ ✓

(g) $x^2 = 8(y+1)$
$4a = 8$
$a = 2$ ✓
focus: $(0, 1)$ ✓

(h) $|2x-1| > 8$
$2x-1 > 8$ $\quad 2x-1 < -8$
$2x > 9$ $\quad 2x < -7$
$x > \frac{9}{2}$ and $x < -\frac{7}{2}$
✓ ✓

Question 12

(a)(i) $\frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{1}{2\sqrt{x}}$ ✓

(ii) $\frac{d}{dx}\left(\tan e^{2x}\right) = 2e^{2x}\sec^2 e^{2x}$ ✓ ✓

(iii) $\frac{d}{dx}\left(4^x\right) = \frac{d}{dx}\left(e^{\ln 4^x}\right)$ ✓ $= \ln 4 e^{\ln 4^x}$
$= \ln 4 . 4^x$ ✓

(b)(i) let $\angle BDC = \theta$
$\cos\theta = \frac{6^2+3^2-5^2}{2(6)(3)}$ ✓
$\theta = 56.25°$ or $56°15'$
$\theta = 56°$ ✓

(ii) $\angle ABD = 180 - 56$ (cointerior $\angle$s add to 180°, AB ∥ CD) ✓
$= 124°$ ✓

(c) $2^4 = 5x+1$ ✓
$5x = 15$
$x = 3$ ✓

(d)(i) $\frac{(2x-1)^4}{2(4)} + c = \frac{1}{8}(2x-1)^4 + c$ ✓ ✓

(ii) $\left[-e^{-x}\right]_0^{\ln 2}$ ✓ $= -e^{-\ln 2} - -e^{-0}$
$= -\frac{1}{2} + 1$
$= \frac{1}{2}$ ✓

Question 13

(a)(i) $m_{AC} = \frac{4-0}{7-1}$

$= \frac{2}{3}$ ✓

(ii) $\tan\theta = \frac{2}{3}$

$\therefore \theta = 34°$ ✓

(iii) $y - 0 = \frac{2}{3}(x-1)$

$2x - 3y - 2 = 0$ ✓

(iv) $D = \left(\frac{1+7}{2}, \frac{0+4}{2}\right)$

$= (4,2)$ ✓

(v) $m_{BD} = \frac{8-2}{0-4}$

$= -\frac{3}{2}$ ✓

$m_{AC} \times m_{BD} = -1$

(vi) $AC = \sqrt{(7-1)^2 + (4-0)^2}$

$= 2\sqrt{13}$

$BD = \sqrt{(8-2)^2 + (0-4)^2}$

$= 2\sqrt{13}$ ✓

$A = \frac{1}{2} \times 2\sqrt{13} \times 2\sqrt{13}$

$= 26u^2$ ✓

(vii) $E = (8,-4)$

(b)(i) In ΔPBQ and ΔCQR

$BQ = CR$ (given)

$\angle PBQ = \angle QCR = 90°$ (square)

$BF = QC$ (length $AB - PA =$ $BC - BQ$ ∵ square, $PA = BQ$) ✓

$\therefore \Delta PBQ \equiv \Delta CQR$ (SAS or RHS) ✓

(ii) let $\angle BQP = \alpha$

$\angle BPQ = 90 - \alpha$ ($\angle$ sum Δ)

$\angle CQR = 90 - \alpha$ (corresp. $\angle$s in cong. Δs) ✓

$\angle BQP + \angle PQR + \angle CQR = 180°$

($\angle$s on st. line) ✓

$\therefore \alpha + \angle PQR + 90 - \alpha = 180°$

$\underline{\angle PQR = 90°}$

(c) $y = 2\sec x$

$y^2 = 4\sec^2 x$

$V = \pi \int y^2 dx$

$= \pi \int_0^{\frac{\pi}{3}} 4\sec^2 x \, dx$ ✓

$= 4\pi \left[\tan x\right]_0^{\frac{\pi}{3}}$ ✓

$= 4\pi\left(\tan\frac{\pi}{3} - \tan 0\right)$

$= \underline{4\sqrt{3}\pi u^3}$ ✓

Question 14

(a)(i) $-\frac{b}{a} = \frac{3}{2}$ ✓

(ii) $\frac{c}{a} = -2$ ✓

(iii) $\frac{1}{\alpha} \times \frac{\beta}{\beta} + \frac{1}{\beta} \times \frac{\alpha}{\alpha} = \frac{\beta+\alpha}{\alpha\beta} = -\frac{3}{4}$ ✓

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{25}{4}$ ✓

(b) LHS $= \sec x(\sec x + \tan x)$

$= \frac{1}{\cos x}\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$

$= \frac{1+\sin x}{\cos^2 x}$ ✓

$= \frac{1+\sin x}{(1+\sin x)(1-\sin x)}$ ✓

$= \frac{1}{1-\sin x}$

$=$ RHS.

(c)(i) $\frac{-\sin x}{\cos x}$ ✓ $= -\tan x$

(ii) $A = \int_0^{\frac{\pi}{4}} \tan x - x \, dx$ ✓

$= \left[-\ln(\cos x) - \frac{x^2}{2}\right]_0^{\frac{\pi}{4}}$ ✓

$= \left(\ln \frac{1}{\sqrt{2}} - \frac{\pi^2}{32}\right) - \left(-\ln 1 - 0\right)$

$= \underline{\frac{1}{2}\ln 2 - \frac{\pi^2}{32}}$ ✓ or $\ln\sqrt{2} - \frac{\pi^2}{32}$

$\int_1^5 \log_e x\,dx \doteq \frac{3-1}{6}\left(0 + 4\log_e 3 + \log_e 5\right)$ ✓

$= \frac{1}{3}(6.00)$

$= 4.00$ ✓

(d)(i) $m_{OP} = \frac{y}{x}$, $m_{PA} = \frac{y}{x-2}$

$m_{OP} \times m_{PA} = -1$

$\therefore \frac{y}{x} \times \frac{y}{x-2} = -1$ ✓

$y^2 = -x^2 + 2x$ ✓

(ii) $x^2 - 2x + 1 + y^2 = 1$ ✓

$(x+1)^2 + y^2 = 1$

centre $(-1, 0)$ radius $= 1$ ✓

(d) $-12 = 27a - 81 + b$ (turning pt.)

$27a + b = 69$

$y' = 3ax^2 - 18x$ (minimum) ✓

$0 = 27a - 54$

$\therefore a = 2$ ✓

$b = 69 - 54$

$b = 15$ ✓

Question 15

(a) $5\pi = 6\theta$

$\theta = \frac{5\pi}{6}$ ✓

$A_{AOB} = \frac{1}{2}(6)^2 \frac{5\pi}{6}$

$= 15\pi \text{ cm}^2$ ✓

(b) (i) geometric ✓

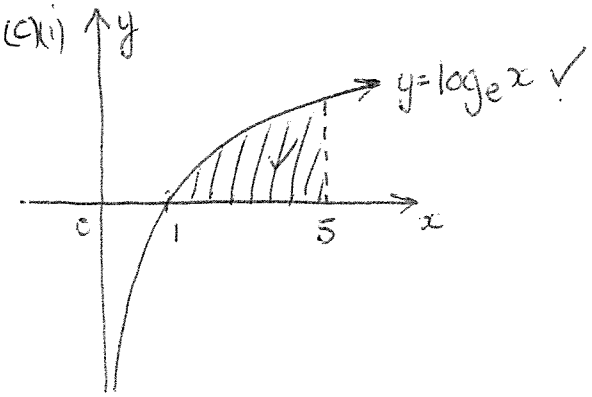
(ii) $\left(4 + 3 + 2\frac{1}{4} + \ldots\right) + \left(3 + 2\frac{1}{4} + \ldots\right)$ ✓

$S_\infty = 4 + 2\left(\frac{3}{1 - 3/4}\right)$

$= 4 + 24$

$= 28\text{m}$ ✓

(c)(i)



✓

(ii)

x	1	3	5
y	0	1.10	1.61

Question 16

(a)(i) $A_1 = 400(1.005) = \$402$ ✓

(iii) $A_2 = (A_1 + 400)1.005$ ✓

$A_2 = (400(1.005) + 400)1.005$

$A_2 = 400(1.005 + 1.005^2)$

$\vdots$

$A_n = 400\left(1.005 + 1.005^2 + \ldots + 1.005^{12n}\right)$

(iii) sub in $n = 8$

$A_8 = 400\left(1.005 + 1.005^2 + \ldots + 1.005^{96}\right)$ ✓

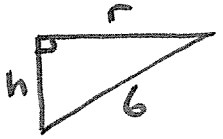
$= \frac{400 \times 1.005\left(1.005^{96} - 1\right)}{0.005}$ ✓

$= \underline{\$49377}$ ✓

(iv) $49377 = P(1.005)^{96}$ ✓

$\underline{P = \$30590}$ ✓

(b)(i) $V = \frac{1}{3}\pi r^2 h$



$h^2 = 36 - r^2$

$h = \sqrt{36 - r^2}$ ✓

$V = \frac{1}{3}\pi r^2\left(\sqrt{36 - r^2}\right)$ ✓

(ii) $\frac{dV}{dr} = \left(36-r^2\right)^{\frac{1}{2}} \times \frac{2}{3}\pi r$ ✓

$+ \frac{1}{3}\pi r^2 \times \frac{1}{2}\left(36-r^2\right)^{-\frac{1}{2}} \times -2r$ ✓

$= \frac{2\pi r\sqrt{36-r^2}}{3} - \frac{\pi r^3}{3\sqrt{36-r^2}}$ ✓

(iii) $\frac{dV}{dr} = 0, \quad \frac{2\pi r\sqrt{36-r^2}}{3} = \frac{\pi r^3}{3\sqrt{36-r^2}}$

$\therefore \quad 2\left(36-r^2\right) = r^2$

$3r^2 = 72$

$r = 2\sqrt{6}$ ✓

test:

r	0	$2\sqrt{6}$	5
$\frac{dV}{dr}$	>0	$=0$	<0

✓

$\therefore$ maximum at $r = 2\sqrt{6}$

$V = \frac{1}{3}\pi \times \left(2\sqrt{6}\right)^2 \times \sqrt{36-\left(2\sqrt{6}\right)^2}$

$V = 8\pi\sqrt{12}$ or $16\pi\sqrt{3}\ u^3$ ✓